How Well Do Doodle Polls Do?

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Abstract. Web-based Doodle polls, where respondents indicate their availability for a collection of times provided by the poll initiator, are an increasingly common way of selecting a time for an event or meeting. Yet group dynamics can markedly influence an individual's response, and thus the overall solution quality. Via theoretical worst-case analysis, we analyze certain common behaviors of Doodle poll respondents, including when participants are either more generous with or more protective of their time, showing that deviating from one's "true availability" can have a substantial impact on the overall quality of the selected time. We show perhaps counter-intuitively that being more generous with your time can lead to inferior time slots being selected, and being more protective of your time can lead to superior time slots being selected. We also bound the improvement and degradation of outcome quality under both types of behaviors.

1 Introduction

Online scheduling tools such as Doodle (www.doodle.com) are a popular way of scheduling events or meetings, with Doodle reporting in 2011 that "online scheduling is used by 67 % of the Swiss and 21 % of the rest of the world".[1] More recent data indicate that in 2014 Doodle had over 20 million monthly users worldwide, with more than 17 million polls created in 2013.[2]

In a Doodle poll, the goal of the poll initiator is to determine the most suitable time for an event or meeting. The initiator selects a set of possible meeting times and sends the Doodle poll invitation to the potential participants. Each participant then checks the boxes for the times they are available to meet; with the default Doodle options, full information about the responses is available to both the initiator and all participants.

Figure 1 shows an example of an open yes-no Doodle poll where three participants have each indicated availability for one or two of the six time slots proposed by the poll initiator; a fourth participant can now enter her name and check boxes for her availability. She can easily see previous responses and that

[1] https://en.blog.doodle.com/2011/07/13/.
[2] https://en.blog.doodle.com/2014/01/29/.

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the most popular slot thus far is 1:00 PM on Saturday April 30, 2016, indicated both via the frequency counter at the bottom of the poll, and the boldface number showing the currently most popular time. As seen here, the Doodle algorithm simply recommends the time slot(s) with the most checked boxes, or "yes" responses.

This social choice mechanism employed by Doodle is equivalent to *approval voting*, where each voter in an election chooses to approve or disapprove each of the candidates. In a Doodle poll, the "voters" are the participants and the "candidates" are the time slots.

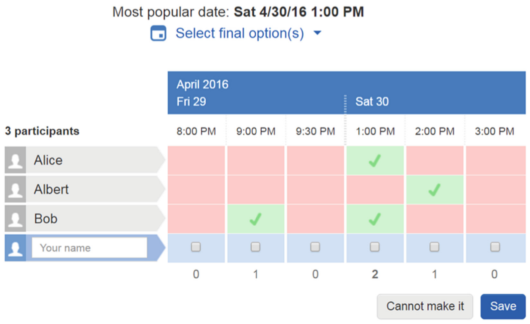

Fig. 1. An example open Doodle poll after three participants have indicated their availability.

While approval voting is the mechanism adopted by a number of professional societies, including the AMS (American Mathematical Society) and the MAA (Mathematical Association of America), such a mechanism clearly has limitations. For one, a voter has no way to express her preference for one candidate she approves over another candidate that she also approves. To be fair, Arrow's classic impossibility theorem has long established that when choosing among three or more candidates, all voting mechanisms have flaws [1]. But approval voting in particular has been a point of controversy, called by Saari and Van Newenhizen [15] a "cure worse than the disease", because, as summarized by [7], "the same voter profile can produce many different results, depending on where each voter decides to draw the line between approved and non-approved candidates." On the other hand, this "feature" of approval voting can be viewed as an advantage, as, according to Brams et al. [3] as interpreted by [7], "it gives each voter 'sovereignty' over the way she expresses her preferences." It is precisely the variation in the location of this "line" drawn by each voter that we model and give a preliminary theoretical analysis for in this work.

We assume each voter has a privately-held, normalized, utility value for each candidate time slot. Intuitively, the utility can be thought of as a quantification of how much the voter expects to benefit from attending the meeting at that time (even if derived simply by satisfying some professional obligation) minus any inconvenience/cost of attending the meeting at that time. To measure the "goodness" of a time slot, we consider the social welfare, or total utility of all voters, for that slot. The fundamental question we ask is, "How well does a Doodle poll work for selecting a time?" We proceed using a standard theoretical worst-case analysis approach common in the algorithms research community.

First we ask, how bad can the time slot chosen by the Doodle mechanism be in comparison to the time slot with maximum social welfare? We show that if an event organizer wishes to maximize social welfare in selecting a time slot, they naturally should not choose one with few “yes” votes. But we also show that, perhaps counter-intuitively social welfare of the chosen slot can be low if people are being very generous with their time and are therefore voting yes too readily. We further show that when voters are protective of their time, voting “no” on slots for which they are available (i.e., slots for which they have utility above the typical voter’s “yes”-threshold), the social welfare can worsen in some cases, while improving in others. We define the notion of a positive (resp. negative) *welfare impact factor* and bound the positive (resp. negative) welfare impact factor of voting protectively. We also show that when voters are cooperative, voting “yes” on slots for which they are not easily available (i.e., slots for which they have a utility below the typical “yes”-threshold), the welfare can also both improve in some cases, while worsening quite dramatically in others. We also provide bounds on the the positive (resp. negative) welfare impact factor of voting cooperatively.

1.1 Related Work

Doodle polls are just one of the group scheduling tools available, and previous research has studied these more generally, considering the conditions under which they are used and useful, and the implications thereof [9,12,16].

There has been extensive research done in approval voting dating back to the 1970s. For surveys on approval voting from the voting theory literature see the book by Brams and Fishburn [2] and the article by Weber [17]. We note that while many researchers have accepted for decades that strategic and manipulative voting behavior is “inevitable” and have continued to seek to quantify the negative effects of it [4], even with respect to approval voting in particular [5,10], others have long argued that the notion of self-interested voting in any large-scale election is implausible, since the act of voting itself is “irrational” [8,14]. In contrast to these large-scale political elections, Doodle polls are usually conducted on a small scale; a sample of over 340,000 polls from the US in a three-month period in 2011 had a median of about 5 respondents and 12 time slots [19], so it is fair to assume that strategic voting indeed takes place.

A recent work of Zou et al. analyzes real Doodle poll data and demonstrates that indeed, Doodle poll participants seem to vote strategically. They hypothesize and give positive evidence for a theory of “social voting” where voters are more likely to say yes to popular time slots, perhaps in an effort to be cooperative [19]. Our model does not attempt to address the “social voting” behavior of the voters. Instead we present a simple model that focuses on the aspect of Doodle polls where some participants generally lean toward being more generous with their time and others lean toward being more protective of their time.

Reinecke et al. [13] also analyze anonymized Doodle poll data, this time from countries around the world, and showed that voting behavior is indeed informed by cultural norms and societal expectations, which supports our model’s notion of an externally-imposed default “yes”-threshold value.

More recently, Obraztsova et al. [11] model the Doodle poll as a game, where players have utilities for each slot, similar to our model. Their paper focuses instead on identifying and showing that trembling hand perfect Nash equilibria (under the assumption that voters derive a utility bonus when they act cooperatively) behave consistently with the "social voting" theory [19] of Zou et al. And in an earlier work, Xu [18] proposes the use of auctions as an alternative to Doodle polls for selecting a good time slot, citing the benefit of allowing participants to specify a valuation for each slot in an auction setting, as well as the tendency of participants to give false or incomplete information in Doodle polls.

One way to quantify the effects of strategic voting is to use welfare as a metric. Lehtinen [6] studies the welfare of approval voting outcomes using a simulation-based approach, concluding that the percentage of simulated voting games where the welfare-maximizing candidate is chosen is rather high, whether voters are sincere or utility-maximizing. While our work also uses social welfare to measure the effects of voting behaviors, our methodology is purely via theoretical worst-case analysis. And rather than assuming the traditional utility-maximizing voters, we consider what happens when sincere voters either vote cooperatively (a la the social voting model of Zou et al. [19]) or are more protective of their time.

2 Theoretical Framework

2.1 Formalizing a Yes-No Doodle Poll

We first formalize the activities encompassed in a yes-no Doodle poll, generally following the notation of [19]. A poll initiator creates a poll with a set of time slots, namely $A = \{a_1, a_2, \ldots, a_m\}$, for consideration. The poll is then made available to the n participants or *voters* in a given poll, denoted by $V = \{v_1, v_2, \ldots, v_n\}$. Each voter's *response* or *vote* is a binary vector r_i for voter i, over the m time slots in A, with $r_i(a) = 1$ if voter v_i approves slot a, and $r_i(a) = 0$ otherwise. When it is clear from the context, we use *vote* to either refer to the full vector, or to the binary value the voter assigns to a specific time slot. A vote of 1 is considered a *yes* vote, and a vote of 0 is a *no* vote.

We thus define a Doodle poll instance to be a 4-tuple $I = (A, V, U, R)$, where A is the set of time slots, V is the set of voters, U is the matrix of utility values each voter has for each time slot, and R is the response matrix of votes that each voter enters for each time slot. In this work, we assume for simplicity that yes and no are the only options for voters. While Doodle does have an "if-need-be" option than can be added, the empirical data of [13,19] provided by Doodle on polls from July-September 2011 contained very few three-option polls [19]. Their dataset likewise showed that the vast majority of polls were *open*, where voters can see the responses of participants who have already responded, as opposed to *closed*, where only the poll initiator can see the responses.

Let $s(j) = \sum_{i=1}^{n} r_i(j)$, or the total count of yes votes for slot j, be the reported *score* for a slot a_j. Note that in Doodle, all voters are given equal consideration; there is no weighting of the votes. The default Doodle algorithm is simply to determine the one or more slots which maximize the total reported score, that is $\max_{j \in A} s(j)$. Thus, Doodle may report multiple maximum-score time slots, and the poll initiator then ostensibly chooses among those slots. (While the poll initiator is of course free to choose a slot with a lower score, in an open poll, which is by far the most commonly-used kind [19], the participants can all see which slots have the most votes, so it is reasonable to assume that the poll initiator will generally choose among the slots recommended by Doodle.) Doodle provides no tie-breaking mechanism, but human poll initiators may certainly have biases (e.g. preferring slots selected by board members or senior personnel; time slots that are personally convenient; the earliest time slot; etc.), and so we assume that when there is a tie, any of the tied slots may be selected.

2.2 Valuations and Voter Types

We now consider the assumptions we make about how a voter determines his or her vote. We assume that for each time slot a_j a voter v_i has a utility u_{ij} with $0 \leq u_{ij} \leq 1$ indicating her valuation of attending the meeting or event during that time slot. This utility value may be thought of as somehow representative of or derived from how much monetary value a voter would place on attending the event at a given time.

We assume that there is a *yes-threshold* $0 \leq t \leq 1$ that represents the utility beyond which a voter "typically" votes yes, so each participant or voter v_i is expected to say yes (i.e. $r_i(a_j) = 1$) to a time slot a_j when her utility for that slot $u_{ij} \geq t$. We note that we are assuming this typical yes-threshold is an externally-imposed or socially-determined global value for all voters. Incorporating the possibility of individual default yes-thresholds t_i for each voter i is a direction for future work.

Notice that unlike with Doodle polls, in approval voting, regardless of where a person chooses to "draw their line" as long as they are voting *sincerely* (never voting "no" on one candidate while simultaneously voting "yes" on a less preferred candidate), they are considered to be voting honestly. Whereas in Doodle polls, there is some notion and expectation that the participants will not only be sincere, but also be "forthcoming" about their "true" availability.

Indeed, other studies have often assumed that the most straightforward, "honest" behavior of a voter is simply to vote "yes" on those time slots for which she is available, and "no" on those she is unavailable. However, we note that availability is not so black and white, and in theory, people can make themselves available for *any* time slot, at varying degrees of cost. Our model accounts for the fact that a person's degree of "availability" is in fact on a continuum. For example, if one wished to consider negative utilities (for time slots where costs outweigh the benefits of attending), then a natural yes-threshold would be at utility 0. In this interpretation of the model, the voting behavior of the players can be seen as "honest" (when they vote yes if and only if their utility is positive), or "dishonest" (when they either vote no for positive-utility time slots, or vote yes on negative-utility slots). Re-scaling utilities to the interval $[0, 1]$ allows the previous threshold value of 0 to also be accordingly mapped to a value t in the interval $[0, 1]$.

On the other hand, if the community culture or larger social/societal expectations imply a different "default" threshold t for voting yes on a time slot, where here we think of t as the utility threshold beyond which a participant is "ordinarily expected" to agree on a time slot, then our model still applies. Normalizing the utility values so that they lie between 0 and 1 and using some non-specified default threshold t makes our model general enough to capture multiple interpretations of the utility values and voting.

We assume all participants are *sincere* in their completion of polls, i.e., if v_i says yes to a time slot a_j which has utility u_{ij} then they also say yes to all slots a_k with utility $u_{ik} > u_{ij}$. Note that the social voting hypothesis arising from empirical data analyzed by Zou et al. [19] supports the expectation of sincere participants. (See their Proposition 2.)

Yet in reality some people are either more protective of their time, voting no on a slot even when their utility for it is above the yes-threshold t, or more cooperative, voting yes on a slot even when their utility is below the yes-threshold t. While our analysis does not assume that a poll is open or closed, there are certainly plausible reasons why either variant could lead voters to be protective or cooperative. Note that while such terms may sometimes have associated positive or negative connotations, we merely use them to categorize participants, and no judgment of the voters' behavior is intended.

We define an *ordinary* voter to be one who votes according to the yes-threshold t, as expected, voting yes to exactly the time slots a_j for which her utility $u_{ij} \geq t$, and no to all others. It might be helpful to think of ordinary voters as those who are responding "honestly" in some sense, akin to how other works have discussed a participant's "availability" in a black and white way [13,19]. But the term ordinary more impartially allows our model to apply to the idea that one's availability is on a continuous spectrum and t is the threshold beyond which social convention dictates one should respond yes.

We define a *cooperative* voter to be one who agrees to slots that are below the yes-threshold t, ostensibly trying to make more slots viable options at one's own expense. Since we assume voters are sincere, this is in practice the same as the voter "lowering" the value of the yes-threshold t for her votes. So she effectively uses a different threshold $t' < t$ such that she says yes to a time slot a_j if and only if her utility $u_{ij} \geq t'$.

We define a *restrictive* voter as one who votes no on slots that are above the yes-threshold, perhaps trying to be more protective of her time. Due to our assumption of sincerity, this is equivalent to the voter "raising" the value of the yes-threshold t for her votes. So she uses an alternative threshold $t' > t$ such that she says yes to a time slot a_j when her utility $u_{ij} \geq t'$.

2.3 Analysis Model

We now present the metric we use for the overall quality of each time slot as well as the framework we use for our analysis of the effects of the above-defined voting tendencies.

The *social welfare* of a given slot a_j is $u(a_j) = \sum_{i=1}^{n} u_{ij}$, the total utility assigned to that slot by all voters. Note that the social welfare is a measure of the theoretical goodness of a time slot; it does not account for the actual attendance of the participants, who may ultimately not attend a time slot for which they had voted yes, or may in fact attend at a time slot for which they had voted no.

We use $OPT(I)$ to denote a slot which maximizes the social welfare in a given Doodle poll instance I, and $u(OPT(I))$ to denote the utility (welfare) of an optimal slot. Hence

$$OPT(I) = \arg\max_{a_j \in A} \sum_{i=1}^{n} u_{ij}, \qquad u(OPT(I)) = \max_{a_j \in A} \sum_{i=1}^{n} u_{ij}.$$

Let $DDL(I)$ likewise denote a time slot returned by the default Doodle algorithm, and let $u(DDL(I))$ denote the utility (welfare) of $DDL(I)$.

In the spirit of worst-case analysis, the conventional approach of the theoretical algorithms (and algorithmic game theory) communities, we aim to determine a quantity that captures how far from optimal the Doodle poll mechanism may be. We therefore define the *welfare approximation ratio* of an algorithm DDL for choosing a time slot to be the maximum over all possible Doodle poll instances of the ratio $u(OPT(I))/u(DDL(I))$. I.e., if $\mathcal{I}$ is the set of all possible Doodle poll instances, the welfare approximation ratio of the default Doodle algorithm DDL is

$$\max_{I \in \mathcal{I}} \frac{u(OPT(I))}{u(DDL(I))}.$$

We also consider in this work the effect of cooperative and restrictive voting on welfare. To quantify this effect, we again employ worst-case analysis. Let a partial Doodle poll instance I' be just the first three elements (A, V, U) from the 4-tuple of a complete Doodle poll instance. Let $\mathcal{I}'$ be the set of all partial instances, and let $R_O(I')$ denote the response matrix that results from a given partial instance $I' \in \mathcal{I}'$ when all voters are ordinary. Let $\mathcal{R}_\mathcal{C}(I')$ and $\mathcal{R}_\mathcal{S}(I')$ be the set of all possible response matrices when a positive number of voters are cooperative and restrictive, respectively. (We drop the I' when the instance is clear from context.) Then we define the *positive welfare impact factor* (resp. *negative*) of cooperative voting to be

$$\max_{I' \in \mathcal{I}'} \max_{R_C \in \mathcal{R}_\mathcal{C}} \frac{u(DDL(I', R_C))}{u(DDL(I', R_O))}, \qquad \max_{I' \in \mathcal{I}'} \max_{R_C \in \mathcal{R}_\mathcal{C}} \frac{u(DDL(I', R_O))}{u(DDL(I', R_C))}.$$

Intuitively, these quantities represent the extreme limits of how many times better (resp. worse) social welfare can become when voters are cooperative (as opposed to ordinary).

We define welfare impact factors for restrictive voting analogously. To succinctly specify a partial Doodle poll instance (A, V, U), we use a table such as Table 1a to indicate the utility values of different categories of participants for each of the possible time slots in a Doodle poll.

Table 1. A template for displaying participants' utilities, voter types (ordinary, cooperative, or restrictive), and the number of voters in each group is given in (a), (b) is an example instance using this table format yielding a welfare approximation ratio of $\frac{1}{t} + \frac{n-x}{x}$. (See Lemma 2, below, for more details on (b).)

Participants		Time Slot 1	...	Time Slot m
#	voter type	utility	...	utility
⋮	⋮	⋮	⋮	⋮
#	voter type	utility	...	utility

(a)

Participants		1	2
x	ordinary	t	1
$n-x$	ordinary	0	$t-\epsilon$

(b)

3 Ordinary Voting

We start by evaluating how well the selected slot optimizes social welfare when all participants are ordinary voters. Throughout the examples and analysis, let $\epsilon > 0$ be a fixed constant, which may be arbitrarily small. We begin with an upper bound on the welfare approximation ratio of Doodle, and then we give an instance that demonstrates this upper bound is tight.

Lemma 1. *The welfare approximation ratio of the default Doodle algorithm with only ordinary voters is strictly less than* $\frac{1}{t} + \frac{n-s^*}{s^*}$*, where* s^* *is the score of the winning time slot.*

Proof. Consider any Doodle poll instance, I. We define $s^* = s(DDL(I))$, i.e., the score of the winning time slot, with $1 \leq s^* \leq n$. (We exclude an s^*-value of 0 because that would mean that all voters voted no for every time slot in the poll.) A reported score of s for the slot picked by the algorithm, meaning exactly s^* yes votes, ensures that $u(DDL(I)) \geq s^*t$, since ordinary voters vote yes precisely when their valuation is greater than or equal to t.

Since the time slot OPT was not picked, it must have an equal or smaller reported score than that of DDL. Note that if OPT and DDL had equal reported scores, the poll initiator has no additional information from the poll about voters' utilities for tie-breaking, and thus could have picked either OPT or DDL. Thus, the OPT time slot has at most s^* voters who voted yes (their valuation is at most 1) and the rest, at most $n - s^*$, have valuation strictly less than t. Thus the optimal social welfare is $u(OPT) < s^* + (n - s^*) \cdot t$. Hence, the ratio of the social welfare of OPT compared to the social welfare of the solution selected by Doodle is $u(OPT)/u(DDL) < \frac{s+(n-s^*)t}{s^*t} = \frac{1}{t} + \frac{n-s^*}{s^*}$.

The approximation ratio is largest when t is small, or when $n - s^*$ is large: these observations illustrate some of the inherent limitations of Doodle polls. The first limitation, a poor ratio when the yes-threshold t is small, shows that if people are "too willing" to say yes to a time (perhaps trying to be more agreeable), the chosen slot may be far from the best. Explicitly, as the yes-threshold decreases, the approximation to optimal welfare worsens. The second limitation, that the ratio is largest when $n-s^*$ is large, is perhaps less concerning in practice. When $n-s^*$ is large, that means that s^*, the reported score, is small, and many poll initiators would expect worse results in terms of overall social welfare when the 'most popular' slot has a small number of people voting yes for it.

The delicate dependence on t we have established also points to ways voters may exploit the system, intentionally or unintentionally. Suppose that there are only two slots, and most people value slot a_1 at or just above t, but strongly prefer slot a_2, with a valuation near 1. Most people thus vote yes to both slots. A single person who values both slots at above t but would rather have slot a_1 can now sincerely vote yes for a_1 and no to a_2 to get their preferred slot, harming the social welfare. Alternatively, a different individual could vote yes for their preferred slot a_2, even if their utility for both is below t, causing the slot with the overall better social welfare to be selected.

The formulation of the above proof gives rise to the following instance, showing that the upper bound of Lemma 1 is tight.

Lemma 2. *The welfare approximation ratio of the default Doodle algorithm with only ordinary voters is at least* $\frac{1}{t} + \frac{(n-s^*)(t-\epsilon)}{s^* t}$*, where* $\epsilon > 0$ *may be arbitrarily close to 0.*

Proof. Consider the instance represented in Table 1b. The utility of the first slot is $u(a_1) = xt$, while $u(a_2) = x + (n-x)(t-\epsilon)$. The reported scores with ordinary voters are $s(a_1) = x$ and $s(a_2) = x$, and thus $s^* = x$. Thus, with the tie, either spot may be chosen, and if a_1 is chosen, the indicated ratio is achieved.

While ties such as the instance in Table 1b may in fact be a reality in Doodle polls, if the tie-breaking aspect seems disconcerting, consider that one additional ordinary voter with valuation t for slot a_1 and 0 for a_2 can be added, so that the reported scores are now no longer tied, but the achieved ratios are comparable for sufficiently large n. Likewise, the instance need not have only two time slots; there can be many more slots, all with reported score less than s^*, and lower total social welfare. Since Lemmas 1 and 2 give matching bounds, we have the following theorem.

Theorem 1. *The welfare approximation ratio of the default Doodle algorithm with only ordinary voters is arbitrarily close to* $\frac{1}{t} + n - 1$.

4 Restrictive Voting

In this section, we make the assumption that some subset of the participants of size ℓ are restrictive voters, while the rest, $n-\ell$, are ordinary voters. Though the ℓ must all vote restrictively, they need not have identical valuations. We show that restrictive voting can not only harm, but also improve the social welfare.

4.1 Restrictive Voting Can Improve Social Welfare

We begin by giving an upper bound on the positive welfare impact factor of restrictive voting. Then we demonstrate that this upper bound is tight by providing a lower bound instance showing that restrictive voters can indeed have that degree of positive welfare impact.

Consider an arbitrary instance I' with ℓ restrictive voters. If everyone voted according to the yes-threshold as ordinary voters, slot $a = DDL(I', R_O)$ would be selected. But since the ℓ restrictive voters vote restrictively, slot b is selected. We assume that slots a and b are not the same, since otherwise there is no change in welfare, but make no other assumptions about these slots; they are just two of possibly many. We let $s^*(a)$ indicate the reported score of slot a when all participants are ordinary voters, that is, vote according to the yes-threshold t. Let $s'(a)$ indicate the reported score of slot a when the restrictive voters use an adjusted yes-threshold greater than t.

Fact 1. $u(a) \geq s^*(a)t$.

Proof. When everyone votes according to the yes-threshold, then all yes votes correspond to voters with valuations of at least t.

Fact 2. $u(b) < s^*(b) + (n - s^*(b)) \cdot t$.

Proof. When everyone votes according to the yes-threshold, yes votes correspond to valuations of at most 1, and no votes correspond to valuations strictly less than t.

By Facts 1 and 2, the welfare approximation ratio

$$\frac{u(b)}{u(a)} < \frac{s^*(b) + (n - s^*(b))t}{s^*(a)t}.$$

Since slot a is selected when everyone votes according to the yes-threshold, $s^*(a) \geq s^*(b)$. Suppose that $s^*(a) - s^*(b) = k$ for some fixed constant k. Then

$$\frac{u(b)}{u(a)} < \frac{s^*(a) - k + (n - s^*(a) + k)t}{s^*(a)t} = \frac{1}{t} + \frac{n - s^*(a)}{s^*(a)} - \frac{(1-t)k}{s^*(a)t}.$$

Observe that the second term is largest when $s^*(a)$ is smallest [a perhaps dissatisfying solution to an initiator]. Since k appears only in the final term (and $t \leq 1$), the ratio is largest when k is smallest. If $k = 0$, the ratio is thus $\frac{1}{t} + \frac{n-s^*(a)}{s^*(a)}$. The above discussion gives the following lemma.

Lemma 3. *The positive welfare impact factor of restrictive voters on any instance I' is strictly less than $\frac{1}{t} + \frac{n-s^*}{s^*}$, where s^* is the winning slot score when all voters are ordinary.*

Table 2. Participant types and valuations where restrictive voting improves the social welfare by a factor of $\approx \frac{1}{t} + (n-\ell)/\ell$, $\frac{1}{t}+1$, and $\approx \frac{1}{t}$, respectively, with the last requiring only a single restrictive voter. See Lemmas 4, 14, and 15 for more details.

Participants		1	2
ℓ	restrictive	t	1
$n-\ell$	ordinary	0	$t-\epsilon$

(a)

Participants		1	2
1	restrictive	t	1
$n/2-1$	ordinary	t	1
$n/2$	ordinary	0	$t-\epsilon$

(b)

Participants		1	2
1	restrictive	1	t
$n-1$	ordinary	1	t

(c)

Note that the case where $k = 0$ may be dissatisfying because it involves a tie and tie-breaking procedure. Thus, we also note that when $k = 1$, the ratio becomes

$$\frac{1}{t} + \frac{n - s^*(a)}{s^*(a)} - \frac{1-t}{s^*(a)t} = \frac{1}{t} + \frac{n+1-s^*(a)}{s^*(a)} - \frac{1}{s^*(a)t}.$$

For a matching lower bound, consider the instance illustrated in Table 2a, with valuations identical to that of Table 1b, but now the first group of voters vote restrictively.

Lemma 4. *The positive welfare impact factor of restrictive voting is at least* $\frac{1}{t} + \frac{n-\ell}{\ell}$*, suppressing epsilons.*

Proof. Consider the instance represented in Table 2a. The utilities of the time slots are $u(a_1) = t\ell$ and $u(a_2) = \ell + (n-\ell)(t-\epsilon)$. When all participants vote according to the yes-threshold t, the reported scores are $s^*(a_1) = \ell$ and $s^*(a_2) = \ell$, with the tie meaning either slot can be chosen. When the ℓ restrictive voters vote restrictively, the reported scores are $s'(a_1) = 0$ and $s'(a_2) = \ell$, ensuring that slot a_2 is chosen. Thus, restrictive voting yields a factor $\frac{1}{t} + \frac{n-\ell}{\ell}$ improvement of the social welfare, ignoring epsilons.

Taken together, Lemmas 3 and 4 give the following theorem.

Theorem 2. *The positive welfare impact factor of restrictive voters is* $\frac{1}{t} + n - 1$*.*

Noting that the example in Table 2a requires the poll initiator to select a slot with low reported score when there are few restrictive voters (and indeed that the ratio of $1/t + n - 1$ is only achieved when the winning slot has a score of 1 and most people vote no to both slots), we provide instances in which a single restrictive voter can still have a positive impact on the social welfare, in situations that are more satisfying to a poll initiator.

In Table 2b, the structure is similar to Table 2a with $\ell = 1$, and gives a $\frac{1}{t} + 1$ improvement of the social welfare but now an additional set of ordinary voters ensures that reported scores are at least half the number of participants. While the instance in Table 2b allows a poll initiator to select a time slot for which half of the participants are available, it suffers from the limitation that half of

Table 3. Participant types and valuations where restrictive voting harms the social welfare by a factor of $\approx \frac{1}{t}$, $\approx \frac{1}{t} + \frac{\ell t'}{(n-\ell)t}$, and ℓ, respectively. See Lemmas 6 and 7 for details.

Participants		1	2
1	restrictive	$1-\epsilon$	1
$n-1$	ordinary	1	t

(a)

Participants		1	2
ℓ	restrictive	$t < t' < 1$	0
$n-\ell$	ordinary	1	t

(b)

Participants			1	2	$\cdots$	$n-1$	n	$n+1$
ℓ	1	restrictive	$t+\epsilon$	0	$\cdots$	0	0	t
	1	restrictive	0	$t+\epsilon$	$\cdots$	0	0	t
	$\vdots$	restrictive	0	0	$\ddots$	0	0	t
$n-\ell$	1	ordinary	0	0	$\ldots$	$t+\epsilon$	0	0
	1	ordinary	0	0	$\ldots$	0	$t+\epsilon$	0

(c)

the participants vote no on both slots, yet social expectations may make that an unlikely response for most participants. Thus, we provide a different instance in Table 2c with a single restrictive voter, reported scores indicating all participants are available, and a $1/t$ improvement of the social welfare. Full proofs of these observations are deferred to the Appendix, in Lemmas 14 and 15.

4.2 Restrictive Voting Can Harm Social Welfare

We first provide an instance (Table 3a) showing that a single restrictive voter can harm the welfare by a factor of $\approx \frac{1}{t}$. We assume $t < 1-\epsilon$. The utilities of the time slots are $u(a_1) = n - \epsilon$ and $u(a_2) = (n-1)t + 1$. When all participants vote according to the yes-threshold t, the reported scores are $s^*(a_1) = n$ and $s^*(a_2) = n$, with the tie meaning either slot can be chosen. When the one restrictive voter votes restrictively, the reported scores are $s'(a_1) = n - 1$ and $s'(a_2) = n$, ensuring that slot a_2 is chosen. Thus, restrictive voting decreases the social welfare from $n - \epsilon$ to $(n-1)t + 1$, which for n large is $\approx 1/t$.

Note that the instance in Table 3a is both plausible from a restrictive voter's perspective (choosing to say no to a less preferred slot), and satisfying to a poll initiator (the reported score indicates availability of all participants). We now show that with additional restrictive voters, social welfare can be harmed further, but first provide an upper bound on the negative welfare impact factor of restrictive voting. The proof mirrors that of Lemma 3, and is deferred to the appendix. We then provide a matching lower bound instance in Lemma 6, as portrayed in Table 3b, where $s'(a) = n - \ell$.

Lemma 5. *The negative welfare impact factor of restrictive voters on any instance I' is strictly less than $\frac{1}{t} + \frac{n-s'}{s'}\frac{t'}{t}$, where s' is the reported score with restrictive voters on the slot that wins when all voters are ordinary.*

Lemma 6. *The negative welfare impact factor of restrictive voting is at least* $\frac{1}{t} + \frac{\ell}{(n-\ell)}\frac{t'}{t}$.

Proof. Consider the instance represented in Table 3b. The utilities of the time slots are $u(a_1) = \ell t' + n - \ell$ and $u(a_2) = (n - \ell)t$. When all participants vote according to the yes-threshold t, the reported scores are $s^*(a_1) = n$ and $s^*(a_2) = n - \ell$, so slot a_1 is chosen. When the ℓ restrictive voters vote restrictively, the reported scores are $s'(a_1) = (n - \ell)$ and $s'(a_2) = n - \ell$, so a_2 may be chosen. Thus, restrictive voting decreases the social welfare from $\ell t' + n - \ell$ to $(n - \ell)t$, giving the stated ratio.

We now provide an instance of a different nature exhibiting a negative welfare impact factors linear in the number of restrictive voters, and, when all participants are restrictive voters, that is, $n = \ell$, the corollary is immediate.

Lemma 7. *The negative welfare impact factor of restrictive voting is at least* ℓ*, where* ℓ *is the number of restrictive voters.*

Proof. Consider the instance represented in Table 3c. The utilities of the first n time slots are all equal, with $u(a_1) = \cdots = u(a_n) = t + \epsilon$, while $u(a_{n+1}) = \ell t$. When all participants vote according to the yes-threshold t, the reported scores are $s^*(a_1) = \cdots = s^*(a_n) = 1$ and $s^*(a_{n+1}) = \ell$. When the ℓ restrictive voters vote restrictively, that is, no to slot $n+1$ (but still yes to the slot with valuation $t + \epsilon$), the reported scores are $s'(a_1) = \cdots = s'(a_n) = 1$ and $s'(a_{n+1}) = 0$. Thus, restrictive voting changes the selected time slot from slot $n + 1$ to any of the others, decreasing the social welfare by a factor of ℓ, suppressing epsilons.

Corollary 1. *The negative welfare impact factor of restrictive voting is at least* n*.*

The example in Table 3c has some nice features, but also some limitations. It is certainly possible that a restrictive voter who dislikes most of the time slots, and has similar valuations for two of the slots, may in fact say yes to only one of those slots. However, many poll initiators, when faced with the reported scores when all participants vote restrictively (namely that all slots have reported scores of 0 or 1) are likely to declare none of the options viable rather than selecting a time to which only one participant voted yes. This concern motivates related examples whose details are deferred to Appendix A where the reported score is now a constant fraction of the number of participants (Table 6) and where the effect of restrictive voting depends on the number of time slots (Table 7).

5 Cooperative Voting

In this section, we make the assumption that some subset of the participants of size c are cooperative voters, while the rest, $n - c$, are ordinary voters. Note that the c must all vote cooperatively, but need not have identical valuations. We show that cooperative voting can greatly improve the social welfare, but also can substantially harm it.

Due to space considerations, all proofs in this section are deferred to the appendix.

Table 4. Participant types and valuations where cooperative voting improves the social welfare by a factor of c and $\frac{1}{t} + \frac{c}{n-c}$, respectively. See the proofs of Lemmas 8 and 9 for details.

	Participants		1	2	$\cdots$	$n-1$	n	$n+1$
c	1	cooperative	t	0	$\cdots$	0	0	$t-\epsilon$
	1	cooperative	0	t	$\cdots$	0	0	$t-\epsilon$
	$\vdots$	cooperative	0	0	$\ddots$	0	0	$t-\epsilon$
$n-c$	1	ordinary	0	0	...	t	0	0
	1	ordinary	0	0	...	0	t	0

(a)

Participants		1	2
c	cooperative	$t-\epsilon$	0
$n-c$	ordinary	1	t

(b)

5.1 Cooperative Voting Can Improve Social Welfare

We present some instances of how cooperative voting can help social welfare, many of which arise from slightly altering the valuations (and changing the participant types) of instances illustrating how restrictive voting can harm social welfare. More precisely, by switching the restrictive voters from Table 3c to cooperative, and decreasing valuations of $t+\epsilon$ to t, and those of t to $t-\epsilon$, we get the instance in Table 4a, which gives Lemma 8. In addition, analogously to Corollary 1, when $n = c$, cooperative voting can help social welfare by a factor of n.

Lemma 8. *The positive welfare impact factor of cooperative voting is at least c, the number of cooperative voters.*

We give an instance in Table 4b that exhibits a positive impact factor as detailed in Lemma 9. Notice that when $c = n$ in the instance of Table 4b the welfare improvement factor becomes unbounded. We then give a matching upper bound in in Lemma 10 on the welfare improvement factor of cooperative voting. The proof is analogous to the proof of Lemma 3.

Lemma 9. *The positive welfare impact factor of cooperative voting is at least $\frac{1}{t} + \frac{c}{n-c}$ (suppressing epsilon terms).*

Lemma 10. *The positive welfare impact factor of cooperative voting for any instance I' is strictly less than $\frac{1}{t} + \frac{n-s^*}{s^*}$, where $s^* = s(DDL(I', R_O))$ is the score of the winning slot when all voters are ordinary.*

Let s^* denote the winning slot when respondents are ordinary and vote according to the yes-threshold. Substituting $s^* = n - c$ in Lemma 10 and taking that together with Lemma 9 gives the following theorem.

Theorem 3. *The positive welfare impact factor of cooperative voting is $\frac{1}{t} + \frac{n-s^*}{s^*}$.*

5.2 Cooperative Voting Can Harm Social Welfare

Though cooperative voting may be quite beneficial, it can likewise be quite harmful. As illustrated in the instance in Table 5a, even a single cooperative voter can harm welfare by a factor of $\approx 1/t$. The instance in Table 5b gives Lemma 11, showing the effects of cooperative voting can be even more harmful to social welfare.

Lemma 11. *The negative welfare impact factor of cooperative voting is at least* $1/t'$*, where* $t' < t$ *is the adjusted yes-threshold of the cooperative voters. (This ratio is unbounded when* $t' = \epsilon$*.)*

The same negative welfare impact factor can also be achieved with n cooperative voters, all of whom value slot 1 at 1 and slot 2 at $t' < t$. Note that the situation in Table 5b does not necessarily seem problematic to an initiator selecting a result. The selected slot has reported score of half of the participants, which may in fact be appropriate in some settings. If the tie-breaking aspect is concerning, having one more cooperative participant (with the same valuations) yields essentially the same results. The default yes-threshold t does not play a role in this instance. And while the results are most striking when $t' = \epsilon$ is small, a person who has valuation 0 for one slot and any amount for another slot, no matter how small, may in fact be inclined (socially) to be a cooperative voter, thus saying yes to the slot for which they have some marginal value.

Table 5. Participant types and valuations where cooperative voting harms the social welfare by a factor of $\approx 1/t$ (with a single cooperative voter), $1/t'$, and $\frac{1}{t'} + \frac{n-c}{c}\frac{t}{t'}$, respectively. See the proofs of Lemmas 11 and 12 for details.

Participants		1	2
1	cooperative	0	$t - \epsilon$
$n - 1$	ordinary	1	t

(a)

Participants		1	2
$n/2$	cooperative	0	$t' < t$
$n/2$	ordinary	1	0

(b)

Participants		1	2
c	cooperative	$t' < t$	1
$n - c$	ordinary	0	$t - \epsilon$

(c)

As alarming as Lemma 11 may be, we show in Lemma 12 based on the instance in Table 5c that cooperative voting can have an even more harmful impact. The moral here for Doodle poll participants is perhaps as follows: if you think you are being helpful by voting yes generously in a Doodle poll, don't be so sure: you might actually be making things worse overall.

Lemma 12. *The negative welfare impact factor of cooperative voting is at least* $\frac{1}{t'} + \frac{n-c}{c}\frac{t}{t'}$.

We then upper bound the negative welfare impact factor of cooperative voting in Lemma 13. Observe that the Lemma 12 instance precisely matches the upper bound since $s^* = c$.

Lemma 13. *The negative welfare impact factor of cooperative voters on any instance I' is strictly less than $\frac{1}{t'} + \frac{n-s^*}{s^*}\frac{t}{t'}$, where s^* is the score of the winning slot when all voters are ordinary, i.e., $s^* = s(DDL(I', R_O))$.*

6 Conclusion

People often assume that a Doodle poll is a mechanism for finding the best time slot for a meeting, yet we show in this work that the optimal social welfare is not always achieved. Under ordinary voting, a Doodle-recommended slot may have social welfare $1/t$ times worse than the optimal. This means that we might want voters to (perhaps counter-intuitively) have a higher yes-threshold t. We also show the Doodle-recommended slot may be as bad as $(n - s^*)/s^*$ times worse than the optimal one, where s^* is the score of the winning slot. So, naturally, a winning slot with a large number of yes votes is preferred. We then show that cooperative voters may in fact harm the overall social welfare, while restrictive voters can improve the overall social welfare. In fact, both cooperative and restrictive voting are capable of harming or improving the overall social welfare. We prove worst-case bounds on both the positive and negative welfare impact of both cooperative and restrictive voting in Doodle polls. We find that even with cooperative and restrictive voting, a lower default yes-threshold, while perhaps conventionally thought of as desirable so that the response matrix is more easily filled with yes votes, can in fact be detrimental to the quality of the winning slot.

The impacts on social welfare naturally suggest future work in this area, including the impacts of having both cooperative and restrictive voters in a single poll. Another direction of investigation would be to use an objective function that considers not just total utility of the winning slot but also its number of yes-votes (which presumably predicts the level of attendance at the event). It would also be interesting to incorporate the social voting hypothesis of [19] into our model. An analysis that includes Doodle's "if-need-be" option, though infrequently used, may demonstrate benefits to poll initiators and participants alike, as it allows participants to have more power to express their preferences over the slots, which may result is improved overall social welfare of selected times. Respondents also then have an added ability to appear more cooperative. It would also be interesting to investigate alternate mechanisms that may lead to improved social welfare of the chosen time slot. Additionally, we could ask what tactics the standard game-theoretic utility-maximizing participant could employ in the Doodle game model we have proposed here, and perhaps study the quality of the Nash equilibria outcomes of such a game. Finally, we would like to acquire and experiment with real Doodle data to see how often these welfare impact effects play out. Since we would not have users' private utility values in this case we would have to simulate the utilities and run what-if scenarios to determine how likely and how often we see such effects.

A Appendix

A.1 Restrictive Voting

Lemma 14. *Even if the score of the winning slot must be at least* $n/2$ *and there is only one restrictive voter, the positive welfare impact factor of restrictive voting is still at least* $\frac{1}{t} + 1$ *(suppressing epsilons).*

Proof. Consider the instance represented in Table 2b. The utilities of the time slots are $u(a_1) = nt/2$ and $u(a_2) = n/2 + n(t - \epsilon)/2$. When all participants vote according to the yes-threshold t, $s^*(a_1) = n/2 = s^*(a_2)$, with the tie meaning that slot 1 could be chosen. When the one restrictive voter votes restrictively, still saying yes to slot 2 but now saying no to slot 1, the reported scores become $s'(a_1) = n/2 - 1$ and $s'(a_2) = n/2$, so that slot 2 is now chosen. Thus, suppressing epsilons, the social welfare improves by a factor of $1/t + 1$.

Lemma 15. *Even if the score of the winning slot is* n *and there is only one restrictive voter, the positive welfare impact factor of restrictive voting is still at least* $\frac{1}{t}$*.*

Proof. Consider the instance represented in Table 2c. The utilities of the time slots are $u(a_1) = n$ and $u(a_2) = nt$. When all participants vote according to the yes-threshold t, $s^*(a_1) = n = s^*(a_2)$, with the tie meaning that slot a_2 could be chosen. When the one restrictive voter votes restrictively, still saying yes to slot 1 but now saying no to slot 2, the reported scores become $s'(a_1) = n$ and $s'(a_2) = n - 1$, so that slot 1 is now chosen. Thus, the social welfare improves by a factor of $1/t$.

Proof (of Lemma 5). Mirroring the proof of Lemma 3, define $a = DDL(I', R_O)$ to be the slot selected when everyone votes according to the yes-threshold, and b is the slot selected when the ℓ restrictive voters vote restrictively. Since we are analyzing the negative welfare impact factor, we must upper bound $u(a)/u(b)$. Observe that by the definitions of a and b and that since restrictive voting can only lower reported scores, we have that $s'(a) \leq s'(b) \leq s^*(b) \leq s^*(a)$. With that observation, and noting that, similarly to Fact 1, $u(b) \geq s^*(b)t$, we then have that $u(b) \geq s'(a)t$. Since $t < t'$ in restrictive voting, and a restrictive yes vote indicates a valuation of at most 1, while a restrictive no vote indicates a valuation less than t', Fact 2 now becomes $u(a) < s'(a) + (n - s'(a))t'$. A comparable remaining argument to that of Lemma 3 thus gives the resulting upper bound.

Consider the instance represented in Table 6. Let $k > 2$ be a fixed constant, and without loss of generality, assume $k \mid n$ and $2 \mid n$, for ease of analysis. The instance has $n/2$ restrictive voters with valuations as before, but also $n/2$ ordinary voters, numbered $i = 1$ to $n/2$, who all value slot $n/2 + 1$ at $t - \epsilon$, the slots i to $i + n/k - 1$ (wrapping around for slots exceeding $n/2$) at t, and the rest at 0. The utilities of the time slots are $u(a_1) = \ldots = u(a_{n/2}) = ((n/k) + 1)t + \epsilon$ and $u(a_{n/2+1}) = nt - n\epsilon/2$. When all participants vote according to the yes-threshold t, the reported scores are $s^*(a_1) = \cdots = s^*(a_{n/2}) = n/k + 1$ and

Table 6. Participant types and valuations where restrictive voting harms the social welfare, with a reported score that is a constant fraction k of the participants.

	Participants		1	2	3	$\dots$	$n/2+1$
$n/2$	1	restrictive	$t+\epsilon$	0	0	0	t
	1	restrictive	0	$t+\epsilon$	0	0	t
	$\vdots$	restrictive	0	0	$\ddots$	0	t
	1	restrictive	0	0	0	$t+\epsilon$	t
	$n/2$	ordinary	for ordinary voter $i = 1 \dots n/2$, t for slots i to $i + n/k - 1$, wrapping around; 0 otherwise				$t-\epsilon$

$s^*(a_{n/2+1}) = n/2$. When the restrictive voters vote restrictively, saying yes to their one slot with valuation $t+\epsilon$ but no to the slot with valuation t, the reported scores are $s'(a_1) = \cdots = s'(a_{n/2}) = n/k + 1$ and $s^*(a_{n/2+1}) = 0$. Thus, since $k > 2$, restrictive voting changes the selected time slot from slot $n/2 + 1$ to any of the other slots, decreasing the social welfare from nt to $(n/k+1)t$, suppressing epsilons. Note that while this example does have a more plausible reported score, it does require the number of time slots to be about half of the number of participants.

Table 7. Participant types and valuations where restrictive voting harms the social welfare by a factor of $\approx m$.

Participants	1	2	3	$\cdots$	$m = \lceil\sqrt{n}\rceil + 1$
$\lfloor\sqrt{n}\rfloor$ restrictive	$t+\epsilon$	0	0	0	t
$\lfloor\sqrt{n}\rfloor$ restrictive	0	$t+\epsilon$	0	0	t
$\lfloor\sqrt{n}\rfloor$ restrictive	0	0	$t+\epsilon$	0	t
$\vdots$ restrictive	0	0	0	$\ddots$	t

Lemma 16. *The negative welfare impact factor of restrictive voting is at least $\approx m$.*

Proof. Consider the instance represented in Table 7. The last slot has utility $u(a_m) = nt$, while the other slots have utilities $\lfloor\sqrt{n}\rfloor(t+\epsilon)$, except possibly for slot $m-1$ which may have smaller utility, due to the square root and truncation with the floor operation. When all participants vote according to the yes-threshold t, most of the slots likewise have reported score $\lfloor\sqrt{n}\rfloor$, again with slot $m-1$ possibly lower, and slot m having $s^*(a_m) = n$. When all n restrictive voters vote restrictively, that is, no to slot m, the reported scores of the first $m-1$ slots are unchanged, with most at $\lfloor\sqrt{n}\rfloor$, but slot m now has $s'(a_m) = 0$.

Thus, restrictive voting changes the selected time slot from slot m to one of the earlier ones (except perhaps for $m-1$), decreasing the social welfare from nt to $\lfloor\sqrt{n}\rfloor(t+\epsilon)$, giving the desired result.

Similarly, the instance in Table 7 with restrictive voting can be transformed to an instance showing that cooperative voting can improve social welfare by a factor of $\approx m$ by making all voters cooperative, changing valuations of t to $t-\epsilon$, and valuations of $t+\epsilon$ to t.

A.2 Cooperative Voting

We again define $s^*(a) = s(DDL(I', R_O))$ to be the score of the winning slot in an instance I' when all voters are ordinary. We now let $s'(a)$ indicate the reported score of slot a when the cooperative voters use an adjusted yes-threshold $t' < t$.

Proof (of Lemma 8). Consider the instance represented in Table 4a. The utilities of the time slots are $u(a_1) = \cdots = u(a_n) = t$ and $u(a_{n+1}) = c(t-\epsilon)$. When all participants vote according to the yes-threshold t, the reported scores are $s^*(a_1) = \cdots = s^*(a_n) = 1$ and $s^*(a_{n+1}) = 0$. When the c cooperative voters vote cooperatively, the reported scores become $s'(a_1) = \cdots = s'(a_n) = 1$ and $s'(a_{n+1}) = c$. Thus, cooperative voting changes the selected time slot from any of the first n to time slot $n+1$, increasing the social welfare by a factor of c, suppressing epsilons.

Proof (of Lemma 9). Consider the instance represented in Table 4b. The utilities of the time slots are $u(a_1) = c(t-\epsilon) + n - c$, and $u(a_2) = (n-c)t$. When all participants vote according to the yes-threshold t, the first group of c participants report no for both slots, while the second group report yes for both slots. Thus, $s^*(a_1) = n - c = s^*(a_2)$, with the tie meaning either slot can be chosen. When the c cooperative voters vote cooperatively, they vote yes for slot 1 but still no on slot 2. The ordinary voters are unchanged in their votes. Hence, $s'(a_1) = n$ and $s'(a_2) = n - c$, ensuring that slot a_1 is chosen. The improvement in social welfare when slot a_1 is chosen due to cooperative voters rather than when slot a_2 can be chosen when all voters vote ordinarily is thus a factor of $\frac{1}{t} + \frac{c}{n-c}$ (suppressing epsilon terms).

Proof (of Lemma 10). The proof is analogous to the proof of Lemma 3, except rather than ℓ restrictive voters, we have c cooperative voters. Note that Facts 1 and 2 which lower bound the utility of slot a, the slot that is chosen when everyone is an ordinary voter, and upper bound the utility of slot b, the slot that is chosen when c of the n voters vote cooperatively, still stand as they are established purely on the reported scores of the two time slots when all voters are ordinary. We therefore still have the established upper bound on the welfare approximation ratio of

$$\frac{u(b)}{u(a)} < \frac{s^*(b) + (n - s^*(b))t}{s^*(a)t} < \frac{1}{t} + \frac{n - s^*(a)}{s^*(a)}.$$

Proof (of Lemma 11). Consider the instance represented in Table 5b. If we set $t' = \epsilon$, the utilities of the time slots are $u(a_1) = n/2$ and $u(a_2) = n\epsilon/2$. When all participants vote according to the yes-threshold t, the reported scores are $s^*(a_1) = n/2$ and $s^*(a_2) = 0$. When the first group (half of the participants) vote cooperatively, the reported scores are $s'(a_1) = n/2$ and $s'(a_2) = n/2$. Thus, with cooperative voting, slot a_2 may be chosen instead of a_1. Hence, the utility goes from $n/2$ to $n\epsilon/2$.

Proof (of Lemma 12). Consider the instance represented in Table 5c. The utilities of the slots are $u(a_1) = ct'$ and $u(a_2) = c + (n - c)(t - \epsilon)$. When all participants vote according to the yes-threshold t, the reported scores are $s^*(a_1) = 0$ and $s^*(a_2) = c$. When the c cooperative voters vote cooperatively, the reported scores become $s'(a_1) = c$ and $s'(a_2) = c$. Thus, with cooperative voting, slot 1 may be chosen instead of slot 2, resulting in the indicated change in social welfare.

Proof (of Lemma 13). Mirroring the proof of Lemma 3, define $a = DDL(I', R_O)$ to be the slot selected when everyone votes according to the yes-threshold, and b is the slot selected when the c cooperative voters vote cooperatively. Since we are analyzing the negative welfare impact factor, we must upper bound $u(a)/u(b)$. Observe that since cooperative voters have a lowered threshold of t', the claim paralleling Fact 1 is $u(b) \geq s'(b)t'$. We also know that $s'(b) \geq s^*(a)$ since with cooperative voting there can only be more yes votes than under ordinary voting, so the winning score of b must be at least that of a. Taking these two inequalities together gives us $u(b) \geq s^*(a)t'$. Fact 2 now becomes $u(a) < s^*(a) + (n - s^*(a))t$. A comparable argument to that in the restrictive voting section thus gives the resulting upper bound.

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